

## $V_{us}$ and $m_s$ from hadronic $\tau$ decays

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Recent experimental results on hadronic  $\tau$  decays into strange particles by the OPAL collaboration are employed to determine  $V_{us}$  and  $m_s$  from moments of the invariant mass distribution. Our results are  $V_{us} = 0.2208 \pm 0.0034$  and  $m_s(2 \text{ GeV}) = 81 \pm 22 \text{ MeV}$ . The error on  $V_{us}$  is dominated by experiment, and should be improvable in the future. Nevertheless, already now our result is competitive to the standard extraction of  $V_{us}$  from  $K_{e3}$  decays, and it is compatible with unitarity.

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### INTRODUCTION

Already more than a decade ago it was realised that the hadronic decay of the  $\tau$  lepton could serve as an ideal system to study low-energy QCD under rather clean conditions [1]. In the following years, detailed investigations of the  $\tau$  hadronic width as well as invariant mass distributions have served to determine the QCD coupling  $\alpha_s$  to a precision competitive with the current world average [2, 3]. The experimental separation of the Cabibbo-allowed decays and Cabibbo-suppressed modes into strange particles opened a means to also determine the mass of the strange quark [4–12], one of the fundamental QCD parameters within the Standard Model.

These determinations suffer from large QCD corrections to the contributions of scalar and pseudoscalar correlation functions [1, 12–14] which are additionally amplified by the particular weight functions which appear in the  $\tau$  sum rule. A natural remedy to circumvent this problem is to replace the QCD expressions of scalar and pseudoscalar correlators by corresponding phenomenological hadronic parametrisations [4, 7, 9, 10, 15], which turn out to be more precise than their QCD counterparts, since the by far dominant contribution stems from the well known kaon pole.

Additional suppressed contributions to the pseudoscalar correlators come from the pion pole as well as higher excited pseudoscalar states whose parameters have recently been estimated [16]. The remaining strangeness-changing scalar spectral function has been extracted from a study of S-wave  $K\pi$  scattering [17, 18] in the framework of resonance chiral perturbation theory [19]. The resulting scalar spectral function was also employed to directly determine  $m_s$  from a purely scalar QCD sum rule [20].

Nevertheless, as was already realised in the first works on strange mass determinations from the Cabibbo-suppressed  $\tau$  decays,  $m_s$  turns out to depend sensitively

on the element  $V_{us}$  of the quark-mixing (CKM) matrix. With the theoretical improvements in the  $\tau$  sum rule mentioned above, in fact  $V_{us}$  represents one of the dominant uncertainties for  $m_s$ . Thus it appears natural to actually determine  $V_{us}$  with an input for  $m_s$  as obtained from other sources [4].

Very recently, new results on the  $\tau$  branching fractions into strange particles have been presented by CLEO [21] and OPAL [22]. In addition, the OPAL collaboration also presented an update on the strange spectral function, previously known only from ALEPH [10]. Both, CLEO and OPAL found  $\mathcal{B}[\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau]$  to be significantly higher than the corresponding ALEPH result. The important impact of these improved findings on the determination of  $V_{us}$  and  $m_s$  will be investigated below.

### THEORETICAL FRAMEWORK

The main quantity of interest for the following analysis is the hadronic decay rate of the  $\tau$  lepton,

$$R_\tau \equiv \frac{\Gamma[\tau^- \rightarrow \text{hadrons} \nu_\tau(\gamma)]}{\Gamma[\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau(\gamma)]} = R_{\tau,NS} + R_{\tau,S}, \quad (1)$$

which experimentally can be decomposed into a component with net-strangeness  $R_{\tau,S}$ , and the non-strange part  $R_{\tau,NS}$ . Additional information can be inferred from the measured invariant mass distribution of the final state hadrons. The corresponding moments  $R_\tau^{kl}$ , defined by [23]

$$R_\tau^{kl} \equiv \int_0^{M_\tau^2} ds \left(1 - \frac{s}{M_\tau^2}\right)^k \left(\frac{s}{M_\tau^2}\right)^l \frac{dR_\tau}{ds} = R_{\tau,NS}^{kl} + R_{\tau,S}^{kl}, \quad (2)$$

can be calculated in complete analogy to  $R_\tau = R_\tau^{00}$ . In the framework of the operator product expansion (OPE),

$R_\tau^{kl}$  can be written as [1]:

$$R_\tau^{kl} = 3S_{\text{EW}} \left\{ \left( |V_{ud}|^2 + |V_{us}|^2 \right) \left( 1 + \delta^{kl(0)} \right) + \sum_{D \geq 2} \left( |V_{ud}|^2 \delta_{ud}^{kl(D)} + |V_{us}|^2 \delta_{us}^{kl(D)} \right) \right\}. \quad (3)$$

The electroweak radiative correction  $S_{\text{EW}} = 1.0201 \pm 0.0003$  [24–26] has been pulled out explicitly, and  $\delta^{kl(0)}$  denotes the purely perturbative dimension-zero contribution. The symbols  $\delta_{ij}^{kl(D)}$  stand for higher dimensional corrections in the OPE from dimension  $D \geq 2$  operators which contain implicit suppression factors  $1/M_\tau^D$  [9, 12, 13].

The separate measurement of Cabibbo-allowed as well as Cabibbo-suppressed decay widths of the  $\tau$  lepton [10, 21, 22] allows one to pin down the flavour SU(3)-breaking effects, dominantly induced by the strange quark mass. Defining the differences

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,NS}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = 3S_{\text{EW}} \sum_{D \geq 2} \left( \delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)} \right), \quad (4)$$

many theoretical uncertainties drop out since these observables vanish in the SU(3) limit.

### DETERMINATION OF $V_{us}$

Employing the SU(3)-breaking difference (4), as a first step, we intend to determine  $V_{us}$ . This approach requires a value for the strange mass from other sources as an input so that we are in a position to calculate  $\delta R_\tau^{kl}$  from theory. In the following, we shall use the result  $m_s(2 \text{ GeV}) = 95 \pm 20 \text{ MeV}$ , a value compatible with most recent determinations of  $m_s$  from QCD sum rules [16, 20, 27] and lattice QCD [28–30]. The compilation of recent strange mass determinations is displayed in figure 1. For comparison, in figure 1, we also display  $m_s$  as obtained from our previous  $\tau$  sum rule analysis [4] for the ALEPH data, as well as this work analysing the OPAL data.

Since the sensitivity of  $\delta R_\tau^{kl}$  to  $V_{us}$  is strongest for the (0,0) moment, where also the theoretical uncertainties are smallest, this moment will be used for the determination of  $V_{us}$ . Inserting the above strange mass value into the theoretical expression for  $\delta R_\tau$  [4], one finds

$$\delta R_{\tau,\text{th}} = 0.218 \pm 0.026, \quad (5)$$

where the uncertainty dominantly results from a variation of  $m_s$  within its errors. Employing the above result in eq. (4), together with the experimental findings  $R_{\tau,NS} = 3.469 \pm 0.014$ ,  $R_{\tau,S} = 0.1677 \pm 0.0050$  [22], as well as  $|V_{ud}| = 0.9738 \pm 0.0005$  [31], we then obtain

$$|V_{us}| = 0.2208 \pm 0.0033_{\text{exp}} \pm 0.0009_{\text{th}} = 0.2208 \pm 0.0034. \quad (6)$$

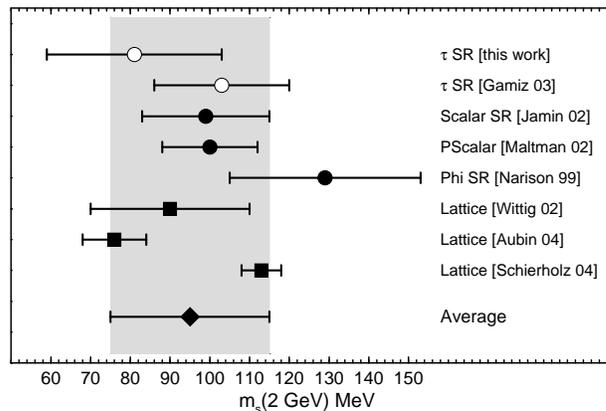


FIG. 1: Summary of recent QCD sum rule [4, 16, 20, 27] and lattice QCD [28–30] results for  $m_s(2 \text{ GeV})$ . For ref. [30] the given error is only statistical.

The first given error is the experimental uncertainty, dominantly due to  $R_{\tau,S}$ , whereas the second error stems from the theoretical quantity  $\delta R_{\tau,\text{th}}$ . For the extraction of  $V_{us}$ , even though the theoretical error on  $\delta R_{\tau,\text{th}}$  is 12%, it represents only a small correction compared to  $R_{\tau,NS}/|V_{ud}|^2$  and thus its error is suppressed. The theoretical uncertainty in  $\delta R_{\tau,\text{th}}$  will only start to matter once the experimental error on  $R_{\tau,S}$  is much improved, possibly through analyses of the BABAR and BELLE  $\tau$  data samples.

One further remark is in order. A sizeable fraction of the strange branching ratio is due to the decay  $\tau \rightarrow K\nu_\tau$ , for which OPAL used the PDG fit result  $\mathcal{B}[\tau \rightarrow K\nu_\tau(\gamma)] = (0.686 \pm 0.023)\%$  [31]. However, this decay can be predicted employing its relation to the decay  $K \rightarrow \mu\nu_\mu(\gamma)$ , which theoretically is known rather well [32, 33]. Updating the numerics of refs. [32, 33], we then obtain  $\mathcal{B}[\tau \rightarrow K\nu_\tau(\gamma)] = (0.715 \pm 0.004)\%$ , much more precise than the experimental value. Adding this result to the remaining strange branching fractions, one finds  $R_{\tau,S} = 0.1694 \pm 0.0049$ , which would lead to  $|V_{us}| = 0.2219 \pm 0.0034$ .

### STRANGE QUARK MASS

Employing the above calculated value for  $V_{us}$ , we are now in a position to determine the strange quark mass  $m_s$  from the SU(3)-breaking difference of eq. (4). Experimentally, various  $(k, l)$  moments have been determined [22]. For low  $k$ , the higher-energy region of the experimental spectrum, which is less well known, plays a larger role and thus in this region the experimental uncertainties dominate the strange mass determination, whereas for higher  $k$  more emphasis is put on the lower-energy region, and there the theoretical uncertainties dominate. At present, the most reliable results for  $m_s$  are obtained from the moments (2, 0) to (4, 0), and we shall only dis-

Parameter	Value	(2,0)	(3,0)	(4,0)
$m_s(M_\tau)$		93.2	86.3	79.2
$R_{\tau,NS}^{kl}$	[22]	+5.1 -5.4	+3.6 -3.7	+2.8 -2.9
$R_{\tau,S}^{kl}$	[22]	-30.9 +23.3	-19.5 +15.8	-13.9 +11.6
$ V_{us} $	$0.2208 \pm 0.0034$	+21.7 -29.8	+14.6 -18.7	+10.6 -13.0
$\mathcal{O}(\alpha_s^3)$	$2 \times \mathcal{O}(\alpha_s^3)$ no $\mathcal{O}(\alpha_s^3)$	-4.0 +4.6	-5.3 +6.5	-6.1 +7.8
$\xi$	$\frac{1.5}{0.75}$	+2.7 +2.3	+4.7 -0.2	+6.3 -2.2
$\alpha_s(M_\tau)$	$0.334 \pm 0.022$	+0.7 +0.1	-0.7 +1.3	-1.6 +2.2
$\langle \bar{s}s \rangle / \langle \bar{u}u \rangle$	$0.8 \pm 0.2$	-8.7 +7.7	-9.9 +8.9	-10.6 +9.6
$f_K$	$113 \pm 2 \text{ MeV}$	-1.8 +1.7	-1.4 +1.4	-1.2 +1.1
Total		+33.6 -44.3	+25.0 -29.5	+21.3 -23.0

TABLE I: Central results for  $m_s(M_\tau)$  extracted from the different moments, as well as ranges for the main input parameters and resulting uncertainties for  $m_s$ .

cuss these here.

The analysis proceeds in complete analogy to our previous work [4]. In table I, we show a detailed account of our results. The first row displays the values of  $m_s(M_\tau)$  obtained from the different moment sum rules and central values for all input parameters. In the following rows, we have listed those input parameters which dominantly contribute to the uncertainty on  $m_s$ , the ranges for these parameters used in our analysis and the resulting shift in  $m_s$ . Only those parameters have been included in the list which at least for one moment yield a shift of  $m_s$  larger than 1 MeV. Finally, in the last row, we display the total error that results from adding the individual uncertainties in quadrature.

Taking a weighted average of the strange mass values obtained for the different moments we then find

$$\begin{aligned} m_s(M_\tau) &= 84 \pm 23 \text{ MeV}, \\ \Rightarrow m_s(2 \text{ GeV}) &= 81 \pm 22 \text{ MeV}, \end{aligned} \quad (7)$$

where the uncertainty corresponds to that of the (4,0) moment. The dominant theoretical uncertainties in the result of eq. (7) originate from higher order perturbative corrections as well as the SU(3)-breaking ratio of the quark condensates  $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle$  [34] which arises in the dimension-4 contribution to eq. (4). A detailed discussion of all input parameters can be found in ref. [4].

In our previous analysis [4], based on the ALEPH data [10], it was observed that  $m_s$  displayed a strong dependence on the number of the moment  $k$ , decreasing with increasing  $k$ , and it was speculated that this behaviour could be due to missing contributions in the higher energy region of the spectrum [35]. With the recent CLEO and OPAL data [21, 22], finding a larger branching fraction of the  $K^- \pi^+ \pi^-$  mode, the decrease of  $m_s$  is now much reduced, although still visible. This issue needs to be clarified further once even better data are available.

## SIMULTANEOUS FIT OF $V_{US}$ AND $M_S$

In principle, it is also possible to perform a simultaneous fit to  $V_{us}$  and  $m_s$  from a certain set of  $(k, l)$  moments. As soon as more precise data are available, this will be the ultimate approach to determine  $V_{us}$  and  $m_s$  from hadronic  $\tau$  decays. With the current uncertainties in the data and the question about a monotonous  $k$ -dependence of  $m_s$ , a bias could be present in the method. Furthermore, the correlations between different moments are rather strong and also have to be included on the theory side.

Here, we shall restrict ourselves to a simplified approach where all correlations are neglected. For the simultaneous fit of  $V_{us}$  and  $m_s$ , we employ the five  $R_\tau^{kl}$  moments (0,0) to (4,0) which have also been used in our previous analysis [4]. Performing this exercise, for the central values we find:

$$|V_{us}| = 0.2196, \quad m_s(2 \text{ GeV}) = 76 \text{ MeV}. \quad (8)$$

The expected uncertainties on these results should be smaller than the individual errors in eqs. (6) and (7), but only slightly since the correlations between different moments are rather strong.

The general trend of the fit result can be understood easily.  $m_s$  from the OPAL data turned out lower than our global average  $m_s(2 \text{ GeV}) = 95 \pm 20 \text{ MeV}$  considered in section 3. Thus, also the corresponding  $\delta R_{\tau,th}$  is lower, resulting in a slight reduction of  $V_{us}$ . Furthermore, the moment-dependence of  $m_s$  is reduced in the fit. Nevertheless, leaving a detailed error analysis for a forthcoming publication [36], at present, we consider eqs. (6) and (7) as our central results.

## CONCLUSIONS

Taking advantage of the strong sensitivity of the flavour-breaking  $\tau$  sum rule on the CKM matrix element  $V_{us}$ , it is possible to determine  $V_{us}$  from hadronic  $\tau$  decay data. This requires a value of the strange quark mass as an input which can be obtained from other sources like QCD sum rules or the lattice. The result for  $V_{us}$  thus obtained is

$$|V_{us}| = 0.2208 \pm 0.0034, \quad (9)$$

where the error is largely dominated by the experimental uncertainty on  $R_{\tau,S}$ , and thus should be improvable with the BABAR and BELLE  $\tau$  data sets in the near future. Already now, our result is competitive with the standard extraction of  $V_{us}$  from  $K_{e3}$  decays [37–40] and a new determination from  $f_K/f_\pi$  as extracted from the lattice [41, 42]. The resulting deviation from CKM unitarity then is

$$1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = (2.9 \pm 1.8) \cdot 10^{-3}, \quad (10)$$

being consistent with unitarity at the  $1.6\sigma$  level.

For the strange mass determination, we have used the three moments  $(2, 0)$  to  $(4, 0)$ , with the result

$$m_s(2\text{ GeV}) = 81 \pm 22\text{ MeV}. \quad (11)$$

Our value for  $m_s$  is on the low side of previous strange mass determinations, but certainly compatible with them. It is also on the borderline of being compatible with lower bounds on  $m_s$  from sum rules [16, 43–46].

Finally, we have performed a simultaneous fit of  $V_{us}$  and  $m_s$  to the five moments  $(0, 0)$  to  $(4, 0)$ . Our central values are completely compatible with the central results of eqs. (6) and (7). Anticipating a detailed analysis of the correlations between different moments, these findings should be considered as an indication of the prospects for the future when more precise experimental data will become available.

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