Light Hadron Spectrum, Renormalization Constants and Light Quark Masses with Two Dynamical Fermions *

SPQcdR Collaboration
D. Bećirević[a], P. Boucaud[a], V. Giménez[b], V. Lubicz[cd], G. Martinelli[e], F. Mescia[cf†], S. Simula[d], and C. Tarantino[cd]

[a]Lab. de Physique Théorique (Bât.210), Univ. Paris XI-Sud, Centre d'Orsay, 91405 Orsay-Cedex, France.

[b]Dep. de Física Teòrica and IFIC, Univ. de València, Dr. Moliner 50, E-46100, Burjassot, València, Spain.

[c]Dip. di Fisica, Univ. di Roma Tre, Via della Vasca Navale 84, I-00146 Roma, Italy.

[d]INFN-Sezione di Roma III, Via della Vasca Navale 84, I-00146 Roma, Italy.

[e]Dip. di Fisica, Univ. di Roma "La Sapienza" and INFN-Sezione di Roma, Piazzale A. Moro 2, I-00185 Roma, Italy.

[f]INFN, Lab Nazionali di Frascati, Via E. Fermi 40, I-00044 Frascati, Italy

The results of a preliminary partially quenched ($N_f = 2$) study of the light hadron spectrum, renormalization constants and light quark masses are presented. Numerical simulations are carried out with the LL-SSOR preconditioned Hybrid Monte Carlo with two degenerate dynamical fermions, using the plaquette gauge action and the Wilson quark action at $\beta = 5.8$. Finite volume effects have been investigated employing two lattice volumes: $16^3 \times 48$ and $24^3 \times 48$. Configurations have been generated at four values of the sea quark mass corresponding to $M_{PS}/M_V \simeq 0.6 \div 0.8$.

Lattice QCD calculations of the light hadron spectrum and quark masses have significantly improved in recent years. In the quenched approximation, i.e. ignoring quark vacuum polarization effects, the accuracy has become smaller than 10% and the quenching error remains then the main source of uncertainty. In the present work we explore sea quark effects in light hadron and quark masses, with two dynamical flavors, using the plaquette gauge action and the Wilson quark action at $\beta = 5.8$.

Configurations have been generated by using the LL-SSOR preconditioned Hybrid Monte Carlo [1,2]. We have implemented the Leap-Frog integration scheme, with trajectory length equal to one and time step $\delta t = 4 \cdot 10^{-3}$, and the BiCGStab inversion algorithm with iterated residual $r = 10^{-15}$. The acceptance probability is found to be larger than 80% and the inversion rounding error results to be well under control, being of the order of 10^{-8}. We have verified that reversibility is satisfied at the relative level of 10^{-11}.

By looking at the autocorrelation times of the plaquette and of the pseudoscalar and vector two point correlation functions, we made the conservative choice of selecting configurations separated by steps of 45 trajectories.

We investigated finite size effects by simulating two lattice volumes: $16^3 \times 48$ ($L \sim 1.1 fm$) and $24^3 \times 48$ ($L \sim 1.6 fm$). At the smaller volume 100 configurations have been generated at each of the four values of the sea hopping parameter ($k_{sea} = 0.1535, 0.1538, 0.1540, 0.1541$), corresponding to $M_{PS}/M_V \simeq 0.6 \div 0.8$. At the larger volume, where the simulation is still in progress, we have generated 50 configurations at $k_{sea} = 0.1535, 0.1540$ and 25 configurations at $k_{sea} = 0.1538, 0.1541$.

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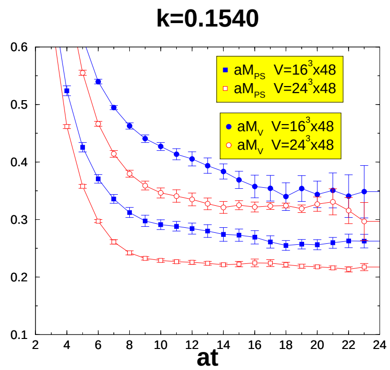

Figure 1. *Degenerate ($k_{v1} = k_{v2} = k_{sea} = 0.1540$) pseudoscalar and vector effective masses, at the two simulated lattice volumes:* $16^3 \times 48$, $24^3 \times 48$.

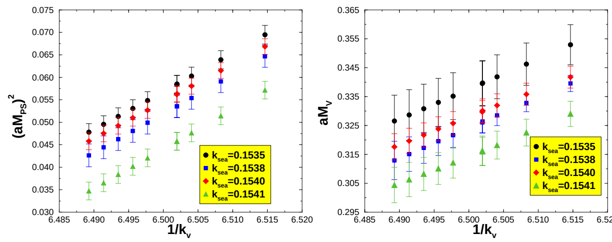

Figure 2. *Pseudoscalar squared(left) and vector(right) meson masses as a function of the valence quark mass. Different symbols refer to different values of the sea quark mass.*

1. Pseudoscalar and vector meson masses

In order to study the light hadron spectrum and quark masses we have calculated two points pseudoscalar and vector correlation functions with both degenerate and non-degenerate valence quarks and with valence quark masses equal or different to the sea quark mass.

We have tried to implement Jacobi smearing [3], but we don't find a significant improvement in the final determination of hadron masses; the main effect is that the ground state can be isolated at smaller times.

The quality of the plateaux of both pseudoscalar and vector meson effective masses is shown in Fig.1. One can also see from the plot that finite volume effects are found to be at the level of $\sim 20\%$ on the smaller volume $V = 16^3 \times 48$. Such large effects are perhaps unexpected since they should scale as $\exp(-ML)$, where $ML \simeq 4.4$ for pseudoscalars and $\simeq 5.8$ for vectors in our simulation on the small lattice. We believe that this point requires further investigations. For the time being, the results we present below follow from the analysis performed at the larger volume $V = 24^3 \times 48$. We observe a smooth enhancement of finite volume effects as the sea quark mass decreases. In Fig.2 left(right) we show the pseudoscalar(vector) meson masses as a function of the valence quark mass ($1/k_v = 1/2(1/k_{v1} + 1/k_{v2})$). Different symbols refer to different k_{sea} values. Within the statistical accuracy, meson masses are found to be linear in the valence quark mass, while the dependence on the sea quark mass is less clear. In particular both pseudoscalar and vector meson masses obtained at $k_{sea} = 0.1538$ are smaller than those at $k_{sea} = 0.1540$, although compatible within the errors. As a consequence, the dependence on the sea quark mass is not easily taken into account in the chiral extrapolations.

In order to determine the lattice spacing and the values of the bare light quark masses we fit the vector meson masses linearly in the squared pseudoscalar masses. Following the method of "lattice physical planes" [4], we set the ratio M_K/M_{K^*} equal to its experimental value, thus obtaining for the inverse lattice spacing the estimate $a^{-1} = 3.0(1)$GeV. For the pseudoscalar and vector meson masses we find $M_\pi = 143(3)$MeV, $M_\rho = 805(19)$MeV (with M_π/M_ρ fixed to its experimental value) and $M_\phi = 983(19)$MeV.

2. Renormalization Constants

The determination of the renormalization constants has been performed non-perturbatively by using the RI-MOM method‡. We have considered the bilinear quark operators $\mathcal{O}_\Gamma = \bar{q}\Gamma q$ with $\Gamma = S, P, V, A, T$ standing respectively for $I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$, and $\sigma_{\mu\nu}$. We also present the result for the renormalization constant of the quark field, Z_q.

Finite volume effects are found to be smaller than 5% for all the renormalization constants and all the values of the sea quark mass.

The dependence of the renormalization constants on the sea quark mass is weak, as one can see form Fig.3, where the scale independent combination Z_P^{RGI} [5] is shown as a function of the renormalization scale.

‡For a more detailed explanation see ref. [5] and references therein.

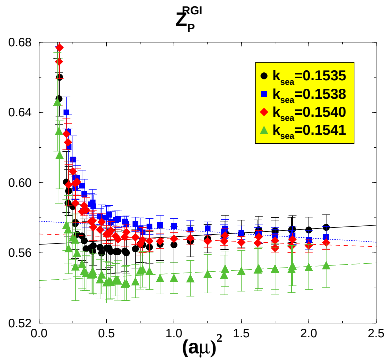

Figure 3. Z_P^{RGI} *at different values of the sea quark mass, as a function of the renormalization scale* $(a\mu)^2$. *At large scales discretization effects introduce a smooth linear dependence on* $(a\mu)^2$.

The results for the renormalization constants, extrapolated to the chiral limit, in the RI-MOM scheme at the scale $\mu = a^{-1}$, read

$$Z_S = 0.72(1)\,, Z_P = 0.55(1)\,, Z_V = 0.69(2)\,,$$
$$Z_A = 0.79(2)\,, Z_T = 0.77(5)\,, Z_q = 0.79(1)\,. \quad (1)$$

3. Light quark masses

In order to determine the values of quark masses we used, as in ref. [6], the standard procedures based on the vector(V) and axial-vector(A) Ward Identities(WI).

The VWI connects the quark mass renormalization constant to the scalar density one. The corresponding mass definition is given by

$$m_q^{(VWI)}(\mu) = Z_S^{-1}(\mu a)\,\frac{1}{2a}\left(\frac{1}{k_q} - \frac{1}{k_{cr}}\right)\,, \qquad (2)$$

where k_q is the Wilson hopping parameter and k_{cr} is its critical value, corresponding to $M_{PS}^2(k_v = k_{sea} = k_{cr}) = 0$. The AWI, instead, relates the quark mass renormalization constant to the renormalization constant of the axial and pseudoscalar operators, leading to the following expression for the renormalized quark mass

$$m_q^{(AWI)}(\mu) = \frac{Z_A}{Z_P(\mu a)}\,\frac{\langle \sum_{\vec{x}} \partial_0 A_0(x) P^\dagger(0)\rangle}{2\langle \sum_{\vec{x}} P(x) P^\dagger(0)\rangle}\,. \quad (3)$$

In order to get the physical values of light quark masses, we study the dependence of the squared pseudoscalar masses on simulated valence and sea quark masses. The observed behaviour on the sea quark mass doesn't allow us to extract from the fit the dependence on (am_{sea}), and the dependence on the valence quark mass is found to be linear. We perform, therefore, a fit to the form

$$(aM_{PS})^2 = A + B(am_{v1}^{(AWI)} + am_{v2}^{(AWI)}) \qquad (4)$$

and a similar expression for the VWI quark mass. The constant term A in eq. (4) is due to $\mathcal{O}(a)-$discretization effects and it is only present in the AWI case. For the VWI case these effects are automatically included in the determination of the critical hopping parameter k_{cr}, implying $A = 0$. The physical values of the average up/down (m_l) and of the strange (m_s) quark masses are then obtained by substituting the experimental pion and kaon masses on the l.h.s. of eq. 4 and the value of the lattice spacing $(a^{-1} = 3.0(1)\text{GeV})$.

Quark mass values are converted from RI-MOM at the renormalization scale $\mu = 1/a$ to $\overline{\text{MS}}$ at $\mu = 2\text{GeV}$ by using RG improved perturbation theory at 4−loop accuracy [7]. The preliminary results read

$$m_l^{VWI} = 4.8(5)\text{MeV}\,, m_s^{VWI} = 111(6)\text{MeV}\,,$$
$$m_l^{AWI} = 4.5(5)\text{MeV}\,, m_s^{AWI} = 103(9)\text{MeV}\,. \quad (5)$$

Simulations at other values of β, k_{sea} and larger volumes are required, in order to study the continuum limit, the dependence on the sea quark mass and finite volume effects.

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REFERENCES

1. S. Duane et al., Phys. Lett. B **195** (1987) 216.
2. S. A. Gottlieb et al., Phys. Rev. D **35** (1987) 2531.
3. C. R. Allton *et al.* [UKQCD Collaboration], Phys. Rev. D **47** (1993) 5128 [arXiv:hep-lat/9303009].
4. C. R. Allton et al., Nucl. Phys. B **489** (1997) 427 [arXiv:hep-lat/9611021].
5. D. Becirevic et al., JHEP **0408** (2004) 022 [arXiv:hep-lat/0401033].
6. D. Becirevic et al. [SPQ(CD)R Collaboration], Phys. Lett. B **558** (2003) 69 [arXiv:hep-lat/0208003].
7. K. G. Chetyrkin and A. Retey, Nucl. Phys. B **583** (2000) 3 [arXiv:hep-ph/9910332].